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# GENERATING INFINITE FAMILIES OF MONOGENIC POLYNOMIALS USING A NEW DISCRIMINANT FORMULA

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Abstract. Recently, Otake and Shaska have given a formula for the discriminant of quadrinomials of the form $f(x) = x^n + t(x^2 + ax + b)$. In their concluding remarks, they ask if a formula can be found for the discriminant of $f(x) = x^n + tg(x)$ when $n > \deg(g) = 3$. Assuming that $f(x) = x^n + tg(x)$ is irreducible, and under certain restrictions on a polynomial related to $g(x)$, in this article we give a formula for the discriminant of $f(x)$, regardless of $\deg(g) \geq 1$. We then use our discriminant formula to generate some new infinite families of monogenic polynomials $f(x) = x^n + tg(x)$ with $n > \deg(g)$, when $g(x)$ is monic and $\deg(g) \in \{2, 3, 4\}$.

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## 1. Introduction

Throughout this article, when we say a polynomial $f(x) \in \mathbb{Z}[x]$ is "irreducible", we mean irreducible over $\mathbb{Q}$. We let $\Delta(f)$ and $\Delta(K)$ denote the discriminants over $\mathbb{Q}$, respectively, of the polynomial $f(x)$ and the number field $K$. If $f(x)$ is irreducible, with $f(\theta) = 0$ and $K = \mathbb{Q}(\theta)$, then we have the well-known equation [1]

$$\Delta(f) = [\mathbb{Z}_K : \mathbb{Z}[\theta]]^2 \, \Delta(K), \tag{1}$$

where $\mathbb{Z}_K$ is the ring of integers of $K$. We say that $f(x)$ is *monogenic* if $\mathbb{Z}_K = \mathbb{Z}[\theta]$, or equivalently from (1), that $\Delta(f) = \Delta(K)$. In this case, $\{1, \theta, \theta^2, \ldots, \theta^{\deg f - 1}\}$ is a basis for $\mathbb{Z}_K$, making computations easier, as in the cyclotomic fields [9]. We see from (1) that if $\Delta(f)$ is squarefree, then $f(x)$ is monogenic, but the converse is false in general. In particular, when $f(x)$ is monogenic and $\Delta(f)$ is not squarefree, it can be difficult to establish that all the square factors of $\Delta(f)$ are, in fact, factors

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of $\Delta(K)$. For a generic polynomial, the first step in the procedure is to derive a workable formula for $\Delta(f)$ in terms of the coefficients and exponents, which itself is not an easy task in general. One known situation is the family of trinomials $f(x) = x^n + ax^m + b \in \mathbb{Z}[x]$ with $0 < m < n$. In this case, the formula

$$\Delta(f) = (-1)^{n(n-1)/2} b^{m-1} \left( n^{n/d} b^{(n-m)/d} - (-1)^{n/d} (n-m)^{(n-m)/d} m^{m/d} a^{n/d} \right)^d, \tag{2}$$

where $d = \gcd(n, m)$, is due to Swan [8]. In 2019, the author [4] gave a formula for the discriminant of an irreducible polynomial of the form

$$f(x) = x^n + A(Bx + C)^m \in \mathbb{Z}[x], \text{ with } n \geq 3 \text{ and } 1 \leq m < n, \tag{3}$$

that was used to construct infinite families of monogenic polynomials under the restriction $\gcd(n, mB) = C = 1$. Also in 2019, Otake and Shaska [6] calculated the discriminant of the polynomial $f(x) = x^n + t(x^2 + ax + b)$, where $t \in \mathbb{Z}$ and $n > 2$. The proof of their discriminant formula required thirteen pages of fairly intense computations, and they did not attempt to address when such polynomials were monogenic or even irreducible. They did, however, ask in their concluding remarks if a discriminant formula could be found for polynomials $f(x) = x^n + tg(x)$, where $t \in \mathbb{Z}$ and $n > \deg(g) = 3$.

**Remark 1.** *We note that there is some overlap with the results in* [4] *and* [6]*. For example, the situation in* [4] *when* $m = 2$ *and* $B = 1$ *in* (3) *is a special case of the polynomials addressed in* [6]*. In addition, when* $m \geq 3$ *in* (3)*, a partial answer for the discriminant of* $f(x) = x^n + tg(x)$ *when* $\deg(g) \geq 3$ *is achieved in* [4].

In this article, with certain restrictions on a polynomial related to $g(x)$, and provided that $f(x)$ is irreducible, we give a formula for the discriminant of $f(x) = x^n + tg(x)$ with $n > \deg(g) \geq 1$, regardless of $\deg(g)$. To derive our formula, we use a shorter and less computationally-intense approach than the one used in [6]. Then, using this discriminant formula, we provide a method for generating some examples of infinite families of monogenic polynomials of this form. These results generalize the work in [4] and give a partial answer to the question of Otake and Shaska concerning a discriminant formula for polynomials of the form $f(x) = x^n + tg(x)$, where $t \in \mathbb{Z}$ and $n > \deg(g) = 3$. More precisely, we prove the following:

**Theorem 1.** *Let* $n$ *and* $k$ *be integers with* $n > k \geq 1$*. Let*

$$f(x) = x^n + tg(x), \text{ where } t \in \mathbb{Z} \text{ and}$$

$$g(x) = a_k x^k + a_{k-1} x^{k-1} + a_{k-2} x^{k-2} + \cdots + a_1 x + a_0 \in \mathbb{Z}[x] \text{ with } a_0, a_k \neq 0.$$

*Define*

$$\widehat{g}(x) := a_k(n-k)x^k + a_{k-1}(n-(k-1))x^{k-1} + \cdots + a_1(n-1)x + a_0 n,$$

*and suppose that*

$$\widehat{g}(x) = \prod_{i=1}^{k} (A_i x + B_i),$$

*where the* $A_i x + B_i \in \mathbb{Z}[x]$ *are not necessarily distinct. If* $f(x)$ *is irreducible, then*

$$\Delta(f) = \frac{(-1)^{\frac{n(n+2k-1)}{2}} t^{n-1} \prod_{i=1}^{k} \left( (-B_i)^n + t \sum_{j=0}^{k} a_j A_i^{n-j} (-B_i)^j \right)}{a_0}.$$

**Remark 2.** *Note that* $f(x)$ *is a trinomial in Theorem 1 when* $k=1$*, and Swan's formula* (2) *with* $m=1$ *is recovered in this situation.*

As an application of Theorem 1, we provide a method in the proof of the following corollary to generate some examples of infinite families of monogenic polynomials of the form $f(x)=x^n+tg(x)$, where $n>\deg(g)$, $g(x)$ is monic and $\deg(g)\in\{2,3,4\}$. The proof requires neither Dedekind's criterion [1] nor the Montes algorithm [5], which are standard methods used for establishing monogeneity. Instead, the corollary is proven using mainly elementary methods, along with two other tools. The first tool is a basic result in algebraic number theory (see Theorem 3), while the second tool is a well-known fact from analytic number theory (see Lemma 1).

**Corollary 1.** *Let* $f(x)$, $g(x)$ *and* $\widehat{g}(x)$ *be as defined in Theorem 1 with* $g(x)$ *monic.*

(1) *For any integer* $n\geq 3$*, there exists* $g(x)$*, with* $\deg(g)=2$*, such that* $f(x)=x^n+tg(x)$ *is monogenic for infinitely many prime values of* $t$*.*
(2) *For any integer* $n\geq 5$ *with* $n\equiv 1 \pmod 4$*, there exists* $g(x)$*, with* $\deg(g)=3$*, such that* $f(x)=x^n+tg(x)$ *is monogenic for infinitely many prime values of* $t$*.*
(3) *For any integer* $n\geq 8$ *with* $n\equiv 2 \pmod 6$*, there exists* $g(x)$*, with* $\deg(g)=4$*, such that* $f(x)=x^n+tg(x)$ *is monogenic for infinitely many prime values of* $t$*.*

All computer computations were done using either MAGMA, Maple or Sage.

## 2. PRELIMINARIES

**Definition 1.** *Let* $p$ *be a prime and let*

$$f(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0\in\mathbb{Z}[x].$$

*We say* $f(x)$ *is* $p$-Eisenstein *if*

$$a_n\not\equiv 0 \pmod p,\qquad a_i\equiv 0 \pmod p \quad \textit{for all } 0\leq i\leq n-1$$
$$\textit{and}\quad a_0\not\equiv 0 \pmod{p^2}.$$

We present some known facts that are used to establish Theorem 1 and Corollary 1.

**Theorem 2.** [3] *(Eisenstien's Criterion) Let* $p$ *be a prime and let* $f(x)\in\mathbb{Z}[x]$ *be* $p$*-Eisenstein. Then* $f(x)$ *is irreducible.*

**Theorem 3.** [2] *Let* $p$ *be a prime and let* $f(x)\in\mathbb{Z}[x]$ *be a monic* $p$*-Eisenstien polynomial with* $\deg(f)=n$*. Let* $K=\mathbb{Q}(\theta)$*, where* $f(\theta)=0$*. Then* $p^{n-1}\mid\mid\Delta(K)$ *if* $n\not\equiv 0 \pmod p$*.*

**Theorem 4.** [3] *Let* $f(x)\in\mathbb{Z}[x]$ *be monic and irreducible with* $\deg(f)=n$*. Let* $f(\theta)=0$ *and* $K=\mathbb{Q}(\theta)$*. Then*

$$\Delta(f)=(-1)^{\frac{n(n-1)}{2}}\mathcal{N}_{K/\mathbb{Q}}\left(f'(\theta)\right).$$

**Lemma 1.** *Suppose that* $h(x)=\prod_{i=1}^{k}(a_ix+b_i)$ *with no repeated factors, where* $a_i$ *and* $b_i$ *are integers with* $\gcd(a_i,b_i)=1$*. Suppose further that, for each prime* $r$*, there exists some* $z\in\left(\mathbb{Z}/r^2\mathbb{Z}\right)^*$ *such that* $h(z)\not\equiv 0 \pmod{r^2}$*. Then there exist infinitely many primes* $p$ *such that* $h(p)$ *is squarefree.*

In Lemma 1, the nonexistence of $z \in \left(\mathbb{Z}/r^2\mathbb{Z}\right)^*$ for which $h(z) \not\equiv 0 \pmod{r^2}$ is called a *local obstruction* at the prime $r$. Because the factors of $h(x)$ are all linear in Lemma 1, it follows that if

$$\phi\left(r^2\right) = r(r-1) \geq 2(r-1) > k,$$

then there exists $z \in \left(\mathbb{Z}/r^2\mathbb{Z}\right)^*$ for which $h(z) \not\equiv 0 \pmod{r^2}$. Hence, only finitely many primes $r$ need to be checked for local obstructions. They are precisely the primes $r$ such that $r \leq (k+2)/2$.

**Remark 3.** *Hector Pasten has pointed out to us (private communication) that Lemma 1 follows unconditionally (without the assumption of the abc-conjecture for number fields) from Theorem 1.1 in [7].*

## 3. The Proof of Theorem 1

*Proof of Theorem 1.* First note that

$$f'(x) = nx^{n-1} + t\left(ka_kx^{k-1} + (k-1)a_{k-1}x^{k-2} + \cdots + a_1\right). \tag{4}$$

Suppose that

$$f(\theta) = \theta^n + tg(\theta) = \theta^n + t\left(a_k\theta^k + a_{k-1}\theta^{k-1} + \cdots + a_1\theta + a_0\right) = 0,$$

so that

$$n\theta^n = -nt\left(a_k\theta^k + a_{k-1}\theta^{k-1} + \cdots + a_1\theta + a_0\right). \tag{5}$$

Then, from (4) and (5), it follows that

$$\begin{aligned}
\theta f'(\theta) &= n\theta^n + t\left(ka_k\theta^k + (k-1)a_{k-1}\theta^{k-1} + \cdots + a_1\theta\right)\\
&= -nt\left(a_k\theta^k + a_{k-1}\theta^{k-1} + \cdots + a_1\theta + a_0\right)\\
&\qquad\qquad\qquad + t\left(ka_k\theta^k + (k-1)a_{k-1}\theta^{k-1} + \cdots + a_1\theta\right)\\
&= -t\left(a_k(n-k)\theta^k + a_{k-1}\left(n-(k-1)\right)\theta^{k-1} + \cdots + a_1(n-1)\theta + a_0n\right)\\
&= -t\widehat{g}(\theta)\\
&= -t\prod_{i=1}^{k}\left(A_i\theta + B_i\right).
\end{aligned}$$

Then, writing $\mathcal{N}$ for the norm $\mathcal{N}_{K/\mathbb{Q}}$, where $K = \mathbb{Q}(\theta)$, and noting that $\mathcal{N}(\theta) = (-1)^n ta_0$, we have

$$(-1)^n ta_0 \mathcal{N}\left(f'(\theta)\right) = (-1)^n t^n \prod_{i=1}^{k}\mathcal{N}\left(A_i\theta + B_i\right).$$

Thus,

$$\mathcal{N}\left(f'(\theta)\right) = \frac{t^{n-1}\prod_{i=1}^{k}\mathcal{N}\left(A_i\theta + B_i\right)}{a_0}. \tag{6}$$

To calculate $\mathcal{N}(A_i\theta + B_i)$, let $z = A_i\theta + B_i$ so that $\theta = (z - B_i)/A_i$. Hence,

$$\begin{aligned}
0 &= A_i^n f(\theta)\\
&= A_i^n\left(\theta^n + tg(\theta)\right)\\
&= A_i^n\left(\left(\frac{z-B_i}{A_i}\right)^n + t\left(a_k\left(\frac{z-B_i}{A_i}\right)^k + a_{k-1}\left(\frac{z-B_i}{A_i}\right)^{k-1} + \cdots + a_0\right)\right)\\
&= (z-B_i)^n + t\sum_{j=0}^{k} a_j A_i^{n-j}(z-B_i)^j,
\end{aligned}$$

from which it follows that

$$\mathcal{N}(z) = \mathcal{N}(A_i\theta + B_i) = (-1)^n\left((-B_i)^n + t\sum_{j=0}^{k} a_j A_i^{n-j}(-B_i)^j\right).$$

Therefore, from (6) and Theorem 4, we conclude that

$$\Delta(f) = \frac{(-1)^{\frac{n(n+2k-1)}{2}} t^{n-1}\prod_{i=1}^{k}\left((-B_i)^n + t\sum_{j=0}^{k} a_j A_i^{n-j}(-B_i)^j\right)}{a_0}. \qquad \square$$

## 4. THE PROOF OF COROLLARY 1

*Proof of Corollary 1.* The strategy for each part is the same. We start with a possible factorization of $\widehat{g}(x)$ and retrofit the coefficients using necessary divisibility conditions. We give the details of this process for part (2), and sketch the proofs for parts (1) and (3), since the methods are similar.

To establish part (2), let $n \geq 5$ be an integer such that $n \equiv 1 \pmod 4$. Since $g(x)$ is monic and $k = \deg(g) = 3$, we have that $g(x) = x^3 + a_2x^2 + a_1x + a_0$ and

$$\widehat{g}(x) = (n-3)x^3 + a_2(n-2)x^2 + a_1(n-1)x + a_0n. \tag{7}$$

Suppose that

$$\begin{aligned}
\widehat{g}(x) &= (x+1)(x+n)((n-3)x + a_0) \qquad (8)\\
&= (n-3)x^3 + \left(n^2 - 2n - 3 + a_0\right)x^2 + \left(n^2 - 3n + a_0n + a_0\right)x + a_0n. \qquad (9)
\end{aligned}$$

Observe that

$$\begin{aligned}
n^2 - 2n - 3 + a_0 &\equiv -3 + a_0 \pmod{n-2} \text{ and}\\
n^2 - 3n + a_0n + a_0 &\equiv 2(a_0 - 1) \pmod{n-1}.
\end{aligned}$$

Thus, by equating coefficients in (7) and (9), we arrive at the system of congruences

$$\begin{aligned}
a_0 &\equiv 3 \pmod{n-2}\\
a_0 &\equiv 1 \pmod{(n-1)/2}
\end{aligned}$$

Therefore, by the Chinese Remainder Theorem, it follows that

$$a_0 \equiv 2n - 1 \pmod{(n-2)(n-1)/2},$$

and so we can write

$$a_0 = s(n-2)(n-1)/2 + 2n - 1. \tag{10}$$

Then, using (10), and again equating coefficients in (7) and (9), yields

$$a_2(n-2) = (n-2)\left(n + 2 + s(n-1)/2\right),$$

from which we conclude that

$$a_2 = n + 2 + s(n-1)/2. \tag{11}$$

Similarly,

$$\begin{aligned}(n-1)a_1 &= n^2 - 3n + (s(n-2)(n-1)/2 + 2n - 1)(n+1)\\ &= (n-1)(3n + 1 + s(n-2)(n+1)/2),\end{aligned}$$

so that

$$a_1 = 3n + 1 + s(n-2)(n+1)/2. \tag{12}$$

Consequently, from (10), (11) and (12), we have that

$$f(x) = x^n + tg(x),$$

where

$$\begin{aligned}g(x) = x^3 &+ (n + 2 + s(n-1)/2)\, x^2\\ &+ (3n + 1 + s(n-2)(n+1)/2)\, x + s(n-2)(n-1)/2 + 2n - 1.\end{aligned}$$

In light of (8) and assuming that $f(x)$ is irreducible, we may apply Theorem 1 with $A_1 = B_1 = A_2 = 1$, $B_2 = n$, $A_3 = n-3$ and $B_3 = a_0 = s(n-2)(n-1)/2+2n-1$ to calculate

$$\Delta(f) = (-1)^{n(n+5)/2} t^{n-1} T_1 T_2 T_3, \text{ where} \tag{13}$$

$$\begin{aligned}T_1 &= (-s(n-3)/2 - 1)\, t + (-1)^n,\\ T_2 &= \left(s\left(n^2 - n\right)/2 - n^2 + n + s - 1\right) t + (-n)^n \text{ and}\\ T_3 &= -Zt + (-1)^n \left(s(n-1)(n-2)/2 + 2n - 1\right)^{n-1}, \text{ with}\end{aligned}$$

$$\begin{aligned}Z = (n-3)^{n-3} \Big(&\left(s^2 - 2s\right) n^3 + \left(-4s^2 + 18s - 4\right) n^2\\ &+ \left(5s^2 - 18s + 4\right) n + \left(-2s^2 - 14s + 124\right)\Big)/4.\end{aligned}$$

At this point, we want to choose, if possible, a value of $s$ so that the product $T_1T_2T_3$ satisfies the hypotheses of Lemma 1 for some infinite set of values of $n$. Computer calculations suggest that $s = 3$ achieves this goal when $n \equiv 1 \pmod 4$. To establish this claim, we let $s = 3$ and proceed as follows. In this situation, we have that

$$g(x) = x^3 + \left(\frac{5n+1}{2}\right) x^2 + \left(\frac{3n^2 + 3n - 4}{2}\right) x + \frac{3n^2 - 5n + 4}{2},$$

and

$$\begin{aligned}T_1 &= \left(\frac{-3n+7}{2}\right) t + (-1)^n,\\ T_2 &= \left(\frac{n^2 - n + 4}{2}\right) t + (-n)^n \text{ and}\\ T_3 &= \left(\frac{-(n-3)^{n-3}\left(3n^3 + 14n^2 - 5n + 64\right)}{4}\right) t\\ &\qquad + (-1)^n \left(\frac{3n^2 - 5n + 4}{2}\right)^{n-1}.\end{aligned}$$

Writing $T_i$ as $a_it+b_i$, we claim that $\gcd(a_i,b_i)=1$ for each $i$. This claim is clearly true for $i=1$, and an easy gcd-argument shows that it is also true for $i=2$. For $i=3$, let $d=\gcd(a_3,b_3)$. Then $d$ divides

$$\left(513n^3+9738n^2+20889n-90432\right)V-(513n+8028)\,U=1179648=2^{17}3^2,$$

where

$$U=(n-3)\left(3n^3+14n^2-5n+64\right) \text{ and } V=3n^2-5n+4.$$

Since $n\equiv 1 \pmod 4$, it is easy to see that $V/2\equiv 1 \pmod 2$. Also, $V/2\equiv 0 \pmod 3$ if and only if $n\equiv 2 \pmod 3$. However, $U/4\equiv 1 \pmod 3$ when $n\equiv 2 \pmod 3$. Consequently, $d=1$.

With $h(t)=\prod_{i=1}^{3}T_i$, so that $k=3$ in Lemma 1, we see that we only have to check the prime $r=2$ for a local obstruction. Since

$$h(t)\equiv\begin{cases} 3(2t+3)^2 & \pmod 4 \quad \text{if } n\equiv 1 \pmod 8\\ 3 & \pmod 4 \quad \text{if } n\equiv 5 \pmod 8,\end{cases}$$

it follows that there is no local obstruction when $n\equiv 1 \pmod 4$ with $r=2$. Thus, since the factors $T_i$ are distinct, we deduce from Lemma 1 that, for each value of $n\equiv 1 \pmod 4$, there are infinitely many prime values of $t$ such that the product $T_1T_2T_3$ is squarefree. Among such prime values of $t$, we can choose infinitely many primes

$$p>a_0=3(n-2)(n-1)/2+2n-1>n, \tag{14}$$

so that $f(x)=x^n+pg(x)$ is $p$-Eisenstein and therefore irreducible. For such a fixed prime $p$, let $f(\theta)=0$ and $K=\mathbb{Q}(\theta)$. Since the product $T_1T_2T_3$ is squarefree, then $\Delta(K)\equiv 0 \pmod{T_1T_2T_3}$. Finally, we deduce from (14) and Lemma 3 that $p^{n-1}\mid\mid\Delta(K)$, and the proof is complete for part (2) of the corollary.

For part (1), we start with the factorization

$$\widehat{g}(x)=(x+n)\left((n-2)x+a_0\right).$$

Using the same procedure as used in part (2) yields

$$a_0=sn-s+1 \text{ and } a_1=s+n-1,$$

so that

$$g(x)=x^2+(s+n-1)x+sn-s+1 \text{ and}$$
$$f(x)=x^n+t\left(x^2+(s+n-1)x+sn-s+1\right).$$

Then the generic discriminant of $f(x)$ in the parameters $s$ and $t$ is

$$\Delta(f)=(-1)^{n(n+3)/2}t^{n-1}T_1T_2,$$

where

$$T_1=(-s+n+1)t+(-n)^n \text{ and}$$
$$T_2=(n-2)^{n-2}(s-n+3)t+(-1)^n(sn-s+1)^{n-1}.$$

Here we let $s=0$, as suggested by computer calculations, and use Theorem 1 to get

$$T_1=(n+1)t+(-n)^n,\quad T_2=(n-2)^{n-2}(-n+3)t+(-1)^n \text{ and}$$
$$\Delta(f)=(-1)^{n(n+3)/2}t^{n-1}\left((n+1)t+(-n)^n\right)\left((n-2)^{n-2}(-n+3)t+(-1)^n\right).$$

With $h(t)=\prod_{i=1}^{2}T_i$, so that $k=2$ in Lemma 1, we see that we only have to check $r=2$ for a local obstruction. Since

$$h(t)\equiv\begin{cases} t & \pmod 4 \quad \text{if } n\equiv 0 \pmod 4\\ (2t+3)^2 & \pmod 4 \quad \text{if } n\equiv 1 \pmod 4\\ 3t & \pmod 4 \quad \text{if } n\equiv 2 \pmod 4\\ 3 & \pmod 4 \quad \text{if } n\equiv 3 \pmod 4,\end{cases}$$

it follows that there is no local obstruction in any case when $r=2$. Hence, $h(t)$ satisfies the conditions of Lemma 1 for any integer $n\geq 3$, and the conclusion of part (1) follows with

$$g(x)=x^2+(n-1)x+1.$$

Finally, for part (3), we start with the factorization

$$\widehat{g}(x)=(x-1)(x+1)(x-n)\left((n-4)x+a_0\right).$$

Using the same procedure used previously, with $s=4$, we get

$$\begin{aligned} a_0&=2n^3-16n^2+38n-27,\\ a_1&=-2n^2+15n-27,\\ a_2&=-2n^3+12n^2-14n-2 \text{ and}\\ a_3&=2n^2-11n+9,\end{aligned}$$

so that

$$\begin{aligned} g(x)=x^4&+\left(2n^2-11n+9\right)x^3\\ &+\left(-2n^3+12n^2-14n-2\right)x^2+\left(-2n^2+15n-27\right)x\\ &+2n^3-16n^2+38n-27.\end{aligned}$$

Then,

$$\Delta(f)=(-1)^{n(n+7)/2}t^{n-1}T_1T_2T_3T_4,$$

where $T_i$ is a linear polynomial in $t$. The exact formulas for the $T_i$ are too large to include here, and the computations to show that the product $T_1T_2T_3T_4$ satisfies the conditions of Lemma 1 are too tedious to include here as well. $\square$

As an illustration of $\Delta(f)$ for part (3) of Corollary 1, we provide the small example $n=8$:

$$\Delta(f)=t^7(78t-1)(106t-1)(5501t+16777216)(1171892480t+125129118027271453).$$

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