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IMPROVING THE TEACHING OF MATHEMATICS TO STUDENTS OF SCIENCE AND ENGINEERING

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Abstract. To improve the teaching of mathematics to students of science and engineering, an holistic approach strongly based on the use of software for symbolic computation with numerical and graphical capabilities is advocated. Within three semesters the quality and quantity of mathematics understood and implementable by those students can be significantly enhanced with this approach, which was tested during an accelerated course that covered all pertinent content within eight weeks.

1. Introduction

In 2010 most instructors of mathematics at the tertiary educational level employ, to some but variable extent for purposes of illustration, software for symbolic computation in their teaching of standard courses such as calculus and linear algebra [1]. Software of this type certainly provides valuable facilities for such purposes through its graphical capabilities, its algebraic operations and its capability of numerical tests over wide ranges. There is, however, much more that is achievable for pedagogical purposes through enlightened use of this software.

When those instructors merely teach the traditional courses in a manner slightly modified to include illustrative demonstrations to students of science and engineering [2], they fail the primary objective of those courses, which is to prepare adequately their students for a technical career that lasts typically five to thirty years after graduation; during all that period the graduate is challenged to apply his mathematical knowledge and expertise to solve problems in his profession. Mathematics is the language of science and technology, but for all subjects other than mathematics the required courses on particular topics in mathematics fall far short of encompassing the full gamut of mathematical knowledge that is desirable for the eventual practice of a technical profession. This enduring and persistent application during a career far transcends the transitory tasks of drill and practice, exercises and examinations on particular topics that occupy the immediate attention of both students and instructors within a particular teaching unit or semester. Moreover, although those illustrations incontestably improve the teaching and learning of mathematical concepts, that software can serve far more effectively in the application to the mathematical component of technical problems during that career, provided that a student becomes satisfactorily acquainted with its capabilities during the learning process. For these reasons we advocate a fundamental reorientation of the teaching of mathematics for it to become based on software for symbolic computation, such that both the learning of concepts and principles and an implementation of those principles reinforces the mathematical capabilities of the student to solve technical problems.

A major obstacle to the application of mathematics as a tool in a technical career is that few academic programmes in subjects other than mathematics include the broad range of mathematical topics that will arise within that technical career. Statistics and data analysis are commonly neglected, and even differential and integral equations are afforded scant coverage in conventional curricula that require as little as three semester courses in mathematics to support biology, chemistry or geology as principal scientific disciplines, with somewhat more for students of physics, engineering or computing. Complex analysis, graph theory, group theory, numerical analysis and vector calculus are mere names that a student of science or engineering might encounter as he or she proceeds through the typically required few courses including differential, integral and multivariate calculus, and perhaps also linear algebra, within the minimum degree requirements for other science subjects. For students that seek to become specialist in mathematics, most degree programs have some vague requirement of other science courses, presumably for cultural reasons, to conform to regulations in a faculty of science, and perhaps some courses in computing; in contrast, for students of science and engineering, courses in mathematics to support their major subject are crucial. Whether a graduate in mathematics becomes employed as a teacher or in industry or commerce, those computer methods can serve as the means to apply the mathematics for whatever purpose. For other science subjects, courses in computing or modeling lack the tradition of mathematics as being an intrinsic component of common degree regulations, but computers will likely play a large and increasing role in the application of mathematics during a technical career.

All these deficiencies can be effectively remedied through the teaching of mathematics based on software for symbolic computation, including extensive graphical and numerical capabilities and an interactive language, in a programme designed holistically to include mathematical topics over a broad range. The duration of such courses within a degree program in science might be as little as two semesters, although realistically three semesters are preferable. The replacing of traditional drill and practice, in particular for aspects of differential and integral calculus and of linear algebra involving manual operations, by the use of computer software enables a great saving of time, which can thus be devoted to increase the range and depth of mathematical topics. Furthermore, the understanding of not only the mathematical concepts and principles but also their implementation with computer software, including the associated limitations, can exceed that in conventional courses, with or without software illustrations, because the details of methods of integration, for instance, can obscure those principles for students who are confronted with the necessity to solve formal mathematical exercises and problems on examinations for the purpose of proceeding through their academic programs in other subjects. Although the use of software for symbolic computation has been much discussed in regard to particular traditional courses, such as calculus, few authors considered implementing the total regiment of mathematics that students in service courses require [3]. In that sense we advocate a possibly radical approach, but one that has already been proved practicable (vide infra).

2. Structure of curriculum

The use of software for symbolic computation inevitably involves learning the language of particular software and thereby becoming acquainted with the programming of computers. Although in the first instance one particular software would likely be encountered, other software for computer algebra has similar design and operation; for that reason, migrating from one software title to another is much less onerous than an initial coming to grips with any particular software, and a careful choice of a particular software to have a gentle learning curve can mitigate the initial barrier. For these reasons we advocate the following approach to teach mathematics with computer software, assuming that a student proceeds through three, likely consecutive, semesters taught with the same software. In the first semester, as a vehicle to become familiar with the basic commands and instructions, the topics include

- arithmetic of integers, real and complex numbers,
- solution of equations and inequalities,
- factoring polynomials,
- elementary functions,
- plots of functions and data in two and three dimensions,
- descriptive geometry,
- trigonometry and
- transformation in complex space,

and differential calculus of a single independent variable includes

- limit,
- derivative,
- explicit and implicit differentiation, and
- differential.

Although much of this material might seem to repeat the content of preceding years of mathematics in school in some environments, an instructor can take advantage of this condition to review those topics, and to explore them from a mature outlook, so to deepen their understanding, while the student learns how to invoke the corresponding operations with the software. Arithmetic can, for example, here include some discussion of number theory and primality, and trigonometry includes consideration of both circular and hyperbolic functions. The first semester of traditional university courses in mathematics is typically concerned mostly with differential calculus; for that reason, at the end of that first semester the student pursuing the new approach is at the same level as with the old approach, although he or she has acquired valuable experience in use of the software as well as reinforcing the understanding of preceding branches of mathematics.

In the second semester, the student proceeds through integral calculus,

- definite integral,
- indefinite integral,
- improper integral,
- numerical quadrature,

with appropriate geometric illustrations and applications of each topic, and multivariate calculus including analytic geometry,

- partial derivative,
- tangent plane and minimization,

- multivariate Taylor and Fourier series,
- exact differential,
- multiple integration,

into linear algebra,

- matrix and determinant,
- matrix inverse and solving linear systems, and
- vector and orthogonality.

The third semester continues with linear algebra in

- eigenvalue and eigenvector,
- vector calculus,
- tensor,
- spreadsheet for mathematical applications,

and progresses through

- ordinary differential equation,
- systems of differential equations of first order,
- partial differential equation,
- integral equation,

into statistical topics,

- probability,
- distribution,
- analysis of variance,
- linear regression,
- non-linear regression, and
- linear and non-linear optimization,

for the treatment of real data. In the process of his or her intensive use of the software, the student becomes, even unconsciously, acquainted with the mechanism of the software, which implies an acquaintance with programming even though little formal programming might at this stage seem to be involved in the use of the current highly sophisticated software for computer algebra. Some basic programming skills are valuable for applications in a technical career; learning a separate language, such as Java or C++, is recommended.

In each of the three semesters, the formal teaching would preferably involve about two hours per week of lecture demonstration of concepts and principles, with another two hours of scheduled and supervised practice in a computer laboratory; further practice and study by the student beyond those hours are naturally expected, for which purpose the software should be generally available in an accessible computer laboratory or on the student's own computer. As the progress through a semester requires an intimate association of students with computers, so must the assessment require the use of a computer, rather than attempting to test any manual skills of the student. An instructor must be aware that, in almost all cases outside the mathematical profession, the practice of mathematics by a scientist or engineer within a few years of graduation has traditionally involved working not manually but instead with extensive consultation of tables of numerical or algebraic content, such as for integrals or differential equations; computer software that is far more powerful than any single reference book or compilation makes such printed tables obsolete.

The advantage of our developed approach is that a student of science and engineering can become proficient in the use of mathematical software to solve technical problems, based on a profound understanding of mathematical and statistical concepts and principles and a practised knowledge of their implementation to solve practical problems. A disadvantage of our approach is that initially a student might have a manual ability less well developed than with traditional courses, but in any case that ability fades rapidly after the completion of particular courses in which it is developed. Another disadvantage might be the additional cost of operating practical sessions, depending on the arrangements of physical and human resources in a particular institution; such analogous laboratory sessions are an accepted and essential component of teaching other science and engineering subjects.

Most mathematicians have been aware of the general ideas discussed above for two or three decades; in that sense the present approach is not novel, but for the teaching of such an integrated course a textbook of appropriate design would be a great asset. Most instructors at university level teach the textbook to a greater or lesser extent, and an assigned textbook has certainly inestimable value to a typical undergraduate student. Many textbooks on particular branches of mathematics, such as calculus or linear algebra, already exist (in printed form!) in which the authors attempt some amalgam of traditional instruction and usage of software. For a course based on computer software, the textbook should, however, have an intrinsically electronic form, but pages or sections could be printed as desired. To prove the practicality of the approach, such a textbook has been developed and published [4]. Even though this book in the form of nine separate computer files is designed particularly for chemistry, the didactic content of those files is nearly all pure mathematics; at those points at which applications or examples in chemistry seem appropriate, advantage is taken of the opportunity to include exercises or illustrations with a chemical or physical theme, such as basic thermodynamic relations in multivariate differential calculus and standard differential equations to treat the kinetics of chemical reactions. In conjunction with the accompanying software, such a textbook is amenable even to self study, thus requiring neither instructor nor classes, and any enterprising student who must suffer under the obsolescent format of traditional courses would do well to supplement his class experience with such a source. As this textbook has been composed from the point of view of a professor of chemistry who seeks to have his students possess not only a broad and profound knowledge of mathematical concepts and principles but also their implementation to solve problems that arise in general, analytical, inorganic, organic, physical and theoretical chemistry, including the treatment of numerical data from teaching and research laboratories, the emphasis has been placed on functionality rather than formality by way of theorems and proofs; recourse to contemporary mathematical textbooks demonstrates that even mathematicians have mostly discarded a purely axiomatic approach to teaching, especially in service courses. The participation in the development of this book by professional mathematicians in various manners has, however, assured that the electronic book has an internal cohesion and a mathematical outlook consistent with almost a standard mathematical point of view. The content of the book has been assembled in the light of the content of traditional textbooks; practically no significant topic of calculus, linear algebra or differential equations that appears in multiple standard textbooks is absent from this electronic book. Numerical aspects, typically neglected in core courses taught by pure mathematicians but essential for practical applications in science and engineering, have been included in a systematic manner.

We endorse the view that, to teach mathematics to students of science and engineering, the instructors should generally be professionally qualified mathematicians, because any student should be exposed to varied points of view from experts in their particular subjects. For the book Mathematics for Chemistry, a mathematician would have adequate scope to teach the mathematics without distraction from chemical or physical digressions, but a student of chemistry could profitably study the chemical examples and undertake the exercises with a chemical context. We present here no example of this approach to the teaching of mathematical topics; the best way to become acquainted with our approach is through scrutiny of the book and the direct operation of its executable commands in a sequence for any selected topic. The printed page here is an inadequate medium to convey the power of this approach, but a few examples are mentioned elsewhere [5].

3. Practical implementation

Although three semesters are considered likely an optimal duration for the teaching and learning of the pertinent material for students of science and engineering, we are unaware of an actual implementation of such a programme, but intensive teaching of the same material has been proved practicable within a smaller period. For instance, a course, of title Mathematical Preparation for Analytical and Physical Chemistry, has been delivered in Universidad de Costa Rica that covered all material outlined above within eight weeks; this course comprised three sessions per week, each of duration four hours, of which the first 75 minutes (on average) was occupied with lecture demonstration of the mathematical concepts and principles and their implementation; the remaining time was devoted to supervised practice. The prerequisite for this course entailed the equivalent of integral calculus but not multivariate calculus. According to this regimen, during the first four weeks the mathematics was formally a review of what the students had already encountered; within this period the objective was to have the students become familiar with the provided software (Maple 13) [6]. During the next four weeks new mathematics was introduced, namely multivariate calculus, linear algebra, differential and integral equations and statistical topics, as outlined above. The criterion for successful completion of this course was stated to be satisfactory fulfillment of 80 per cent of the assigned exercises, which numbered 230 in total, so averaging ten per period. Some exercises consisted of applications to chemistry, but most were purely mathematical in nature, designed to reinforce the concepts discussed during the lecture and described at sufficient length in the assigned worksheets extracted from the textbook [4]; some exercises comprised a single part devoted to the solution of a particular problem by algebraic, numerical or graphical means, and other exercises with multiple parts were designed to explore various aspects of a particular topic through selected examples. For no student was the period of four hours sufficient for the solution of these exercises; because most students had no other course or formal activity during this summer season, they were able to allocate whatever additional time was necessary for the work, either in the computer laboratory of the course or on computers elsewhere to which they had access. The response to this course by the students was unanimously positive: they felt not only that they had learned much useful mathematics that they could apply in whatever chemical studies or technical activities might follow, but also, and more importantly, that they had sufficiently mastered both the concepts and the software to an extent that enabled them to implement therewith the mathematical operations to solve whatever technical problems might arise. The students would naturally rely on various electronically stored materials, including the textbook, for additional guidance, just as students of traditional courses rely on reference books and tables. A subsequent survey, after one semester, of students who completed this course verified that they applied the software in their study of chemistry and enhanced their understanding of the chemical topics through an improved facility with the mathematical underpinning. The circumstances under which this course operated were atypical: even though we recommend three semesters as an optimal duration of the total program of content, the fact of the actual delivery within eight weeks and the emphatic success of this course demonstrate the creative possibilities that teaching and learning with computer software open.

Some universities in Europe operate on a block system whereby students devote their attention to a single course for a few weeks. To cover all the content of the above lists, this system is obviously applicable to teach the mathematics with the available software and textbook in one, two or three such blocks.

4. Conclusion

To prepare for a productive technical career in this era of the computer whether in developed or developing countries, students of science and engineering need more and better education in mathematics and statistics than what they have been receiving within the traditional required courses. Symbolic computation within a program designed to encompass a broad range of topics can provide not only that improved and expanded learning within a similar number of semester courses but also a means to enhance greatly the capability to solve technical problems. Departments of mathematics that fail to respond to these conditions are abdicating their responsibility to provide timely and effective mathematical education for students of science and engineering.

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