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# A PARALLEL ALGORITHM FOR ANALYTICAL SOLVING OF PARTIAL DIFFERENTIAL EQUATIONS SYSTEMS.

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Abstract. There is produced a parallel algorithm for symbolic solving systems of partial differential equations by means of multivariate Laplace-Carson transform. There is considered a system of n equations with m as the greatest order of partial derivatives and right hand parts of a special type. Initial and conditions are input. As a result of Laplace-Carson transform of the system according to initial condition we obtain an algebraic system of equations. A method to obtain compatibility conditions is discussed.

## 1. Introduction

The Laplace and Laplace–Carson transform is useful in many problems of solving differential equations (for example [1, 2, 4]) It reduces a system of partial differential equations to an algebraic linear system with polynomial coefficients. Parallel algorithms for solving such systems are being developed actively (for example, [5, 3]). It enables to construct parallel algorithms for solving linear partial differential equations with constant coefficients and systems of equations of various order, size and types. The application of Laplace–Carson transform permits to obtain compatibility conditions in symbolic way for many types of PDE equations and systems of PDE equations.

## 2. Problem statement

Denote $\widetilde{m} = (m_1, \ldots, m_n)$. Consider a system

$$\sum_{k=1}^{K} \sum_{m=0}^{M} \sum_{\widetilde{m}} a^j_{\widetilde{m}k} \frac{\partial^m}{\partial^{m_1} x_1 \ldots \partial^{m_n} x_n} u_k(x) = f_j(x), \tag{1}$$

where $j = 1, \ldots, K$, $u_k(x)$, $k = 1, \ldots, K$, — are unknown functions of $x = (x_1, \ldots, x_n) \in \mathbf{R}^n_+$, $f_j \in S$, $a^j_{\widetilde{m}k}$ are real numbers, $m$ is the order of a derivative, and $k$ –the number of an unknown function. Here and further summing by $\widetilde{m} = (m_1, \ldots, m_n)$ is executed for $m_1 + \ldots + m_n = m$.

We consider all input functions reducible to the form;

$f_j(t) = f^i_j(x)$, $x^i_j < t < t^{i+1}_j$, $i = 1, \ldots, I_j, x^1_l = 0, t^{I_j+1}_j = \infty$,
where

$$f^i_j(t) = \sum_{s=1}^{S^i_j} P^i_{js}(t) e^{b^i_{js} t}, \quad i = 1, \ldots, I_j, \quad j = 1, \ldots, k, \tag{2}$$

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and $P_{js}^i(x) = \sum_{l=0}^{L_{js}^i} c_{sl}^{ji} x^l$.

Denote by **A** a class of functions which are reducible to the form (2).

We solve a problem with initial conditions for each variable. Introduce notations for them. Denote by $\Gamma^\nu$ a set of vectors $\gamma = (\gamma_1, \dots, \gamma_n)$ such that $\gamma_\nu = 1$, $\gamma_i = 0$, if $i < \nu$, and $\gamma_i$ equals 0 or 1 in all possible combinations for $i > \nu$. The number of elements in $\Gamma^\nu$ equals $2^{\nu-1}$.

Denote $\beta = (\beta_1, \dots, \beta_n)$, $\beta_i = 0, \dots, m_i$, a set of indexes such that the derivative of $u^k(x)$ of the order $\beta_i$ with respect to the variables with numbers $i$ equals $u_{\beta,\gamma}^k(x^{(\gamma)})$ at the point $x = x^\gamma$ with zeros at the positions $\mu$ for which the coordinates $\gamma_\mu$ of $\gamma$ equal 1. For example, if zeros stand only at the places with the numbers $1, 2, 3$, then $\gamma = (1, 1, 1, 0, \dots, 0)$. Functions $u_{\beta,\gamma}^k(x^{(\gamma)})$ must also belong to **A**. To be short we shall not write down the expressions for $u_{\beta,\Gamma}^k(x^{(\gamma)})$.

The algorithm component is the definition of compatible initial conditions. The system (1) is to be solved under such conditions.

Data file contains the coefficients, the initial conditions and the right-hand members $f_j$,
$l = 1, \dots, K$.

The data for functions $f_j$ consists of the polynomial coefficients, parameters of exponents, the bounds of smoothness intervals.

## 3. Laplace–Carson transform

Consider the space $S$ of functions $f(x)$, $x = (x_1, \dots, x_n) \in \mathbf{R}_+^n$, $\mathbf{R}_+^n = \{x : x_i \geq 0,\ i = 1, \dots, n\}$, for which $\mathcal{M} > 0, a = (a_1, \dots, a_n) \in \mathbf{R}^n$, $a_i > 0$, $i = 1, \dots, n$, exist such that for all $x \in \mathbf{R}_+^n$ the following is true: $|f(x)| \leq \mathcal{M} e^{ax}$, $ax = \sum\limits_{i=1}^{n} a_i x_i$.

On the space $S$ the Laplace–Carson transform (**LC**) is defined as follows:

$$LC : f(x) \mapsto F(p) = p^1 \int_0^\infty e^{-px} f(x) dx,$$

$$p = (p_1, \dots, p_n), \quad p^1 = p_1 \dots p_n,$$

$$px = \sum_{i=1}^{n} p_i x_i, \quad dx = dx_1 \dots dx_n.$$

LC is performed symbolically at the class **A**.

## 4. Parallel Laplace–Carson algorithm

The steps, at which parallel calculations are possible and reasonable we denote by term **Block**. If indexes are contained, the ways of parallelization are pointed by them.

4.1. **LC of a system.** Let $LC : u^k \mapsto U^k, u_{\beta,\gamma}^k(x^{(\gamma)}) \mapsto U_{\beta,\gamma}^k(p^{(\gamma)}), f_j \mapsto F_j$, the notation $p^{(\gamma)}$ is correspondent to the notation $x^{(\gamma)}$. Denote by $\|\gamma\|$ the "length" of $\gamma$ — the number of units in $\gamma$, $p^{\widetilde{m}} = p_1^{m_1} \dots p_n^{m_n}$.

**Block 10**

The LC of the left-hand side of the system (1) excluding images of initial conditions is written formally.

**Block 1r**

**r** runs trough the set of multiindexes of $u^k_{\beta,\Gamma}(x^\Gamma)$.

Then

$$LC : \frac{\partial^m}{\partial^{m_1} x_1 \dots \partial^{m_n} x_n} u_k(x) \ \mapsto$$

$$p^{\widetilde{m}} U^k(p) \ + \sum_{\nu=1}^{n} \sum_{\beta_\nu=0}^{m_\nu} \sum_{\gamma\in\Gamma^\nu} (-1)^{\|\gamma\|} p_1^{m_1-\beta_1-\gamma_1} \dots p_n^{m_n-\beta_n-\gamma_n} U^k_{\beta,\gamma}(p^{(\gamma)}).$$

Denote

$$\Phi^j_{mk} = \sum_{\widetilde{m}} a^j_{\widetilde{m}k} \sum_{\nu=1}^{n} \sum_{\beta_\nu=0}^{m_\nu} \sum_{\gamma\in\Gamma^\nu} (-1)^{\|\gamma\|} p_1^{m_1-\beta_1-\gamma_1} \dots p_n^{m_n-\beta_n-\gamma_n} U^k_{\beta,\gamma}(p^{(\gamma)}).$$

As a result of Laplace–Carson transform of the system (1) according to initial conditions we obtain an algebraic system relative to $U^k$

$$\sum_{k=1}^{K} \sum_{m=0}^{M} \sum_{\widetilde{m}} a^j_{\widetilde{m}k} p^{\widetilde{m}} U^k(p) = F_j - \sum_{k=1}^{K} \sum_{m=0}^{M} \Phi^j_{mk}, j = 1, \dots, K. \tag{3}$$

**Block 2k**

**k** runs from 1 to $K$.

These blocks performs LC of the right-hand parts of (1). The properties of **A** allow a further parallelization of calculations.

4.2. **Solution of algebraic system. Block 3**

As a result of Laplace–Carson transform of the system (1) according to initial conditions we obtain the algebraic system (3) relative to $U^k$.

Efficient methods of parallel solving such systems are developed (for example [5], [3]).

At this stage the problem of definition of compatibility conditions arises (see blocks 4s,5). With respect to compatible conditions we use the inverse Laplace–Carson transform and obtain the correct solution of PDE system.

4.3. **Compatibility conditions.** Call a rational fraction *"a proper fraction"* if the degree of each variable (over **C**) in numerator is less then its degree in denominator.

Call by the class **B** a set of equations, defined by conditions

• the solutions of algebraic system may be represented as sums of proper fractions with exponential coefficients,

• the denominators of these proper fractions may be reduced to a product of linear functions.

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The class **B** admits symbolic implementation of further calculations.

Denote by $D$ the determinant of the system (3), $D_i$ the maximal order minors of the extended matrix of (3). A case when there is a set $\mathcal{Q}$ of zeros of $D$ with infinite limit point at $\operatorname{Re} p_k > 0$, $k = 1, \dots, n$, is of most interest. Solving the system (1) we obtain $U^k$ as fractions with $D$ in the denominators. The inverse Laplace–Carson transform is possible if $\alpha_k$, $k = 1, \dots, n$, exist such that these functions are holomorphic in the domain $\operatorname{Re} p_k > \alpha_k$. So we make a demand: $D_i$ has zeros at $\mathcal{Q}$ of multiplicity not less than multiplicity of corresponding zeros of $D$. This demand produces requirements to the LC images of initial conditions functions, and after

$\mathrm{LC}^{-1}$ transform – to initial conditions. They turn to be dependent. We obtain the so-called compatibility conditions.

**Block 4s**
s depends upon the number of relations, from which the compatibility conditions arise.

The blocks calculate the values of numerators at zeros of denominators.

**Block 5**

The block implements parallel solving of the system of equations, produced by relations for compatibility conditions.

**Block 6k**

The blocks perform the $\mathrm{LC}^{-1}$ of $U^k$. Note, that the steps of calculation of multivariate $\mathrm{LC}^{-1}$ are produced sequentially.

## 5. Examples

5.1. **Example 1.** To demonstrate the LC algorithm let us consider in details solving of a system of three equations with three unknown functions $f(x,y,z)$, $g(x,y,z)$, $h(x,y,z)$ on $\mathbf{R}^3_+$.

$$\begin{array}{rcl} \frac{\partial}{\partial x}f+\frac{\partial}{\partial z}g+\frac{\partial}{\partial y}h &=& x\\ \frac{\partial}{\partial z}f+\frac{\partial}{\partial x}g+\frac{\partial}{\partial y}h &=& y\\ \frac{\partial}{\partial y}f+\frac{\partial}{\partial x}g+\frac{\partial}{\partial z}h &=& z \end{array} \tag{1}$$

We shall consider the problem when values of unknown function at zeros of $x,y,z$ are taken as initial conditions.

As we have the derivatives of the first order with respect to each variable we need nine initial conditions – three for every unknown function – at $(0,y,z)$, $(x,0,z)$, $(x,y,0)$, correspondingly to the order of the derivative. A requirement is a coincidence of correspondent functions values at the intersection of these planes.

**Block 1r** , r=1,2,3.

Denote values of functions at these points as follows:

$$\begin{array}{lll} f(0,y,z)=f^x, & g(0,y,z)=g^x, & h(0,y,z)=h^x,\\ f(x,0,z)=f^y, & g(x,0,z)=g^y, & h(x,0,z)=h^y,\\ f(x,y,0)=f^z, & g(x,y,0)=g^z, & h(x,y,0)=h^z. \end{array}$$

Denote the images of LC transform of $f,g,h$, respectively by $u,v,w$.

To be transparent in the example we denote LC images of the initial conditions functions by nine various Greek letters $\alpha,\beta,\gamma,\delta,\delta,\varepsilon,\xi,\tau,\sigma$, correspondingly:

| | $(q,r)$ | $(p,r)$ | $(p,q)$ |
|---|---|---|---|
| $f$ | $\alpha$ | $\eta$ | $\delta$ |
| $g$ | $\varepsilon$ | $\xi$ | $\beta$ |
| $h$ | $\tau$ | $\gamma$ | $\sigma$ |

Table 1

In the table the first column points the functions for which the LC images of the initial conditions are considered, the first line indicates the variables upon which these images depend.

**Block 3**

Applying the Laplace-Carson transform to the system (1) we obtain the algebraic system

$$\begin{aligned} pu + rv + qw - p\alpha - r\beta - q\gamma &= \tfrac{1}{p}, \\ ru + pv + qw - r\delta - p\varepsilon - q\gamma &= \tfrac{1}{q}, \\ qu + pv + rw - q\eta - p\varepsilon - r\sigma &= \tfrac{1}{r}. \end{aligned} \tag{2}$$

The solution of this system is

$$u = -\frac{-pq^2 + pqr + qr^2 - r^3}{qr(p-r)(q-r)(p+q+r)} - \frac{(-p^2q^2r + p^2qr^2)\alpha + (-pq^2r^2 + pqr^3)\beta + (pq^2r^2 - q^2r^3)\gamma}{qr(p-r)(q-r)(p+q+r)} - \frac{(pq^2r^2 - qr^4)\delta + (pq^2r^2 - pqr^3)\varepsilon + (-pq^3r + q^3r^2)\eta + (-pq^2r^2 + q^2r^3)\sigma}{qr(p-r)(q-r)(p+q+r)},$$

$$v = -\frac{-p^2q^2 + q^3r + p^2r^2 - qr^3}{pqr(p-r)(q-r)(p+q+r)} - \frac{(p^2q^3r - p^2qr^3)\alpha + (pq^3r^2 - pqr^4)\beta + (p^2q^2r^2 - pq^2r^3)\gamma}{pqr(p-r)(q-r)(p+q+r)} - \frac{(-pq^3r^2 + p^2qr^3)\delta - (p^3q^2r + p^2q^3r - p^3qr^2)\varepsilon}{pqr(p-r)(q-r)(p+q+r)} + \frac{(-p^2q^3r + pq^3r^2)\eta + (-p^2q^2r^2 + pq^2r^3)\sigma}{pqr(p-r)(q-r)(p+q+r)},$$

$$w = \frac{-p^2q + q^2r + p^2r - qr^2}{qr(p-r)(q-r)(p+q+r)} + \frac{(p^2q^2r - p^2qr^2)\alpha + (pq^2r^2 - pqr^3)\beta + (p^2q^2r + pq^3r - pq^2r^2 - q^3r^2)\gamma}{qr(p-r)(q-r)(p+q+r)} + \frac{(-q^2r^3 + p^2qr^2)\delta - (pq^2r^2 - pqr^3)\varepsilon + (-p^2q^2r + q^2r^3)\eta + (-p^2qr^2 + qr^4)\sigma}{qr(p-r)(q-r)(p+q+r)}$$

The determinant $D$ of the system equals

$$D = -(p-r)(q-r)(p+q+r).$$

The bracket $(p+q+r)$ is not important for solving the problem of compatibility – its zeros do not belong to $\mathcal{Q}$.

**Block 4s**, $s = 1, \ldots, 6$.

Consider the sets $p = r,\ q = r$. We demand the numerators of the solutions be zero on these sets. To indicate that the functions of initial conditions are taken for $p = r$ or $q = r$ we use the notations displaced in the following table. If for a function $p = r$ is put we use this function with the index 1, if there is put $q = r$, we use this function with the index 2. To demonstrate the algorithm of getting compatibility conditions display initial conditions and their transformations after substituting of points of $\mathcal{Q}$ into the table.

| | $p=r$ | $q=r$ |
|---|---|---|
| $\alpha(q,r)$ | $\alpha(q,r)$ | $\alpha_2(q,r)$ |
| $\varepsilon(q,r)$ | $\varepsilon(q,r)$ | $\varepsilon_2(r,r)$ |
| $\tau(q,r)$ | $\tau(q,r)$ | $\tau_2(r,r)$ |
| $\theta(p,r)$ | $\theta_1(r,r)$ | $\theta(p,r)$ |
| $\xi(p,r)$ | $\xi_1(r,r)$ | $\xi(p,r)$ |
| $\gamma(p,r)$ | $\gamma_1(r,r)$ | $\gamma(p,r)$ |
| $\delta(p,q)$ | $\delta_1(r,q)$ | $\delta_2(p,r)$ |
| $\beta(p,q)$ | $\beta_1(r,q)$ | $\beta_2(p,r)$ |
| $\sigma(p,q)$ | $\sigma_1(r,q)$ | $\sigma_2(p,r)$ |

Table 2

Substituting $p=r$ and $q=r$ into the numerators of $u$, $v$, $w$, we obtain a system of 6 equations, that connect functions $\alpha,\beta,\gamma,\delta,\delta,\dots,\delta_2$.

$$\left\{\begin{array}{rcl}
-rq^2+2qr^2-r^3-(r^3q^2-qr^4)\alpha+(-q^2r^3+qr^4)\beta_1 & + & \\
+(q^2r^3-qr^4)\delta_1+(q^2r^3-qr^4)\varepsilon & = & 0\\
q^3r-q^2r^2-qr^3+r^4+(q^3r^3-qr^5)\alpha+(q^3r^3-qr^5)\beta_1 & - & \\
-(q^3r^3-qr^5)\delta_1-(q^3r^3-qr^5)\varepsilon & = & 0\\
-q^2r+2qr^2-r^3+(-q^2r^3+qr^4)\alpha+(-q^2r^3+qr^4)\beta_1 & + & \\
+(q^2r^3-qr^4)\delta_1+(q^2r^3-qr^4)\varepsilon & = & 0\\
(pr^4-r^5)\gamma+(pr^4-r^5)\delta_2+(-pr^4+r^5)\eta+(-pr^4+r^5)\sigma_2 & = & 0\\
(p^2r^4-pr^5)\gamma+(p^2r^4-pr^5)\delta_2+(-p^2r^4+pr^5)\eta+(-p^2r^4+pr^5)\sigma_2 & = & 0\\
(-p^2r^3+r^5)\gamma+(-p^2r^3+r^5)\delta_2+(p^2r^3-r^5)\eta+(p^2r^3-r^5)\sigma_2 & = & 0
\end{array}\right.$$

**Block 5**

Solving it with respect to these variables, we get two conditions on them:

$$\begin{array}{rcl}
\alpha & = & -\frac{q-r}{qr^2}-\beta_1+\delta_1+\varepsilon,\\
\gamma & = & -\delta_2+\eta+\sigma_2.
\end{array} \tag{3}$$

We may take arbitrarily all images of initial conditions except of $\alpha$ and $\gamma$ and obtain $\alpha$ and $\gamma$ according to the conditions (3).

For example, we may take the following functions in the table 1.

| | $(q,r)$ | $(p,r)$ | $(p,q)$ |
|---|---|---|---|
| $f$ | $-\frac{1}{r^2}+\frac{2}{qr^2}$ | $\frac{1}{pr}$ | $\frac{1}{p^2q}$ |
| $g$ | $\frac{1}{qr^2}$ | $\frac{1}{p^2r}$ | $\frac{1}{pq}$ |
| $h$ | $\frac{1}{qr}$ | $\frac{1}{pr^2}-\frac{1}{p^2r}+\frac{1}{pr}$ | $\frac{1}{pq^2}$ |

Table 3

The correspondent initial conditions are the follows:

$$f^x=\frac{1}{2}\left(-z^2+2yz\right),\quad g^x=\frac{yz^2}{2},\quad h^x=yz,$$

$$f^y=xz,\quad g^y=\frac{x^2z}{2},\quad h^y=\frac{1}{2}\left(2xz-x^2z+xz^2\right),$$

$$f^z=\frac{x^2y}{2},\quad g^z=xy,\quad h^z=\frac{xy^2}{2}. \tag{4}$$

**Block 6k**, $k = 1, 2, 3$.

Substituting the functions $\alpha, \beta, \gamma, \ldots$ from the table 3 into the solution $u, v, w$, after inverse LC transform we obtain the solution of the system (1) correspondent to the initial conditions (4):

$$f = 1/6(3x^2y - 6xyz + 6yz^2 - 2z^3 - 3(x-z)^2 H(-x+y)H(-x+z)+$$
$$+2(-x+z)^3 H(-x+y)H(-x+z) + 6y(y-z)H(-y+z)+$$
$$+6x(-y+z)H(-y+z) + 2(-y+z)^3 H(-y+z) + 6x(y-z)H(-x+y)H(-y+z)+$$
$$+2(y-z)^3 H(-x+y)H(-y+z) + 6y(-y+z)H(-x+y)H(-y+z));$$
$$g = 1/6(6xy + 6xz - 12xyz + 3z^2 + 3yz^2 - 2z^3 - 3(x-z)^2 H(-x+y)H(-x+z)+$$
$$+2(-x+z)^3 H(-x+y)H(-x+z) + 6y(y-z)H(-y+z) + 6x(-y+z)H(-y+z)+$$
$$+2(-y+z)^3 H(-y+z) + 6x(y-z)H(-x+y)H(-y+z)+$$
$$+2(y-z)^3 H(-x+y)H(-y+z) + 6y(-y+z)H(-x+y)H(-y+z));$$
$$h = 1/6(3xy^2 - 3x^2z - 6yz + 3xz^2 + 6yz^2 - 2z^3 - 3(x-z)^2 H(-x+y)H(-x+z)+$$
$$+2(-x+z)^3 H(-x+y)H(-x+z) + 6y(y-z)H(-y+z) + 6x(-y+z)H(-y+z)+$$
$$+2(-y+z)^3 H(-y+z) + 6x(y-z)H(-x+y)H(-y+z)+$$
$$+2(y-z)^3 H(-x+y)H(-y+z) + 6y(-y+z)H(-x+y)H(-y+z)),$$

where $H(x)$ is the Heaviside step function.

5.2. **Example 2.** The LC algorithm permits to solve equations of various types and of any order if input functions belong to **A**, and the equation is from the class **B**. To demonstrate the application of the algorithm to an equation of the fourth order let us consider the equation of forced vibration of elastic rod:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^4 f}{\partial y} = xy.$$

Initial conditions:

$f(0,y) = a(y)$; $\frac{\partial f(x,y)}{\partial x}\Big|_{x=0} = b(y)$;

$f(x,0) = c(x)$; $\frac{\partial f(x,y)}{\partial y}\Big|_{y=0} = d(x)$;

$\frac{\partial^2 f(x,y)}{\partial y^2}\Big|_{y=0} = g(x)$; $\frac{\partial^3 f(x,y)}{\partial y^3}\Big|_{y=0} = h(x)$

**Block 1** .

$$LC : f(x,y) \mapsto u(p,q),$$

$a(y) \mapsto \alpha(q)$, $b(y) \mapsto \beta(q)$,
$c(x) \mapsto \gamma(p)$, $d(x) \mapsto \delta(p)$,
$g(x) \mapsto \sigma(p)$, $h(x) \mapsto \tau(p)$.

**Block 3**

As a result of LC we obtain the algebraic equation:

$$p^2u - p^2\alpha - p\beta - qu + q\gamma qu + q\gamma = \frac{1}{pq}$$

$D = p^2 + q^2$

Then

$$u = \frac{1 + p^3q\alpha + p^2q\beta + pq^5\gamma + pq^4\delta + pq^3\sigma + pq^2\tau}{pq(p^2+q^4)};.$$

**Block 4s** , $s = 1, 2$.

There are two $Q$ sets of zeros of the denominator

$$D = p^2 + q^2,$$

these sets are defined by the conditions

$p = iq^2$, $p = -iq^2$.

At this sets the numerator of $u$ equals correspondingly

$$A1 = 1 - iq^7\alpha - q^5\beta + iq^7\gamma_1 + iq^6\delta_1 + iq^5\sigma_1 + iq^4\tau_1;$$
$$A2 = 1 + iq^7\alpha - q^5\beta - iq^7\gamma_2 - iq^6\delta_2 - iq^5\sigma_2 - iq^4\tau_2,$$

where

$\gamma_1,\ \delta_1,\ \sigma_1,\ \tau_1$

are the values of functions $\gamma,\ \delta,\ \sigma,\ \tau$ at $p = iq^2$,

$\gamma_1,\ \delta_1,\ \sigma_1,\ \tau_1$

– at $p = iq^2$.

The functions with indexes 1 and 2 depend on different arguments $iq^2$ and $-iq^2$, correspondingly. So it is convenient to take the originals $c, d, g, h$ of $\gamma,\ \delta,\ \sigma,\ \tau$ as data functions of initial conditions and to find $a, b$ as compatible with them. Note that this is a characteristic speciality of equations of such type, for example of elliptic equations.

Solve

$$\begin{cases} A1 &= 0, \\ A2 &= 0 \end{cases}$$

with respect to $\alpha, \beta$.

**Block 5** ,

Compatibility conditions on images of LC:

$$\alpha = -\frac{-q^3\gamma_1 - q^3\gamma_2 - q^2\delta_1 - q^2\delta_2 - q\sigma_1 - q\sigma_2 - \tau_1 - \tau_2}{2q^3};$$

$$\beta = \frac{i(-2i + q^7\gamma_1 - q^7\gamma_2 + q^6\delta_1 - q^6\delta_2 + q^5\sigma_1 - q^5\sigma_2 + q^4\tau_1 - q^4\tau_2)}{2q^5}.$$

Taking concrete functions $c(t), d(t), g(t), h(t)$ of initial conditions, we obtain $a(x)$ and $b(x)$ as compatible with them. In such way we may define, for example, the following compatible initial conditions:

$a = 1 - \frac{x^4}{12},\ b = \frac{x^5}{120}, c = 1 + t^2$.

Finally we obtain the solution satisfying the initial conditions:

$$f(t, x) = 1 + t^2 - \frac{x^4}{12} + \frac{tx^5}{120}.$$

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